Précis of *The Cultural Emergence of Combinatorial Structure*

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Cultural symbol systems, such as language, music, or visual schemas and diagrams, are fundamental to human life, enabling us to communicate, collaborate, and solve problems in wide variety of domains. Language, for example, allows us to describe and coordinate the actions needed to build a shelter or find food. Diagrams and schemas serve as tools to visually represent complex, abstract ideas, facilitating collaboration and problem-solving. Music can evoke deep emotional responses in listeners and create a sense of shared identity. Finally, these systems enable the storage and transmission of knowledge across generations, a fundamental cornerstone of human culture (Deacon, 1998).

Central to the effectiveness of these systems is their *combinatorial structure*. From phonemes in language, to notes in a melody, or the lines in a diagram, these systems combine a finite set of building blocks to express a set of concepts or ideas. This flexible recombination of elements, as illustrated by the versatile use of phonemes in languages to form countless words (Hayes, 2011; Yule, 2022), exemplifies the productivity that is characteristic of combinatorial systems (O'Donnell, 2015).

Research on combinatoriality in signaling systems has evolved from focusing on phonological categories (Liljencrants & Lindblom, 1972) to incorporating computational modeling (de Boer, 2000; Oudeyer, 2005; Zuidema & de Boer, 2009), revealing how discrete categories like vowel systems could emerge through self-organization. Recent research using human subjects in artificial language learning settings suggest that combinatorial structures spontaneously develop even in small systems, emphasizing cognitive biases for pattern recognition and structural preferences (Del Giudice, Kirby, & Padden, 2010; Verhoef, 2012; Roberts, Lewandowski, & Galantucci, 2015; Eryilmaz & Little, 2017).

The central goal of this thesis is to provide a comprehensive account of the evolution of combinatorial symbol systems, particularly by addressing key open questions: identifying underlying cognitive capacities and examining how they interact with fundamental constraints in language evolution. These constraints include simplicity, communicative utility, and the capacity for iconic representation as an alternative to arbitrary combinatorial forms. Pursuing this requires a highly interdisciplinary approach, weaving together insights from experimental psychology, psycholinguistics, evolutionary science, and computational modeling. Starting point for investigation is a puzzling observation: In human culture, combinatorial structures are predominantly found in language, including speech, gestures, and writing systems, as well as in music. This contrasts with an abundance of non-combinatorial signaling systems in human culture—such as diving signals, traffic signs, sports referee signals, and hazard symbols—where combinatoriality seems feasible but is absent. Why do we not see a more widespread use of combinatorial systems?

To address this, the thesis will argue that combinatorial structure only emerges under a set of very specific conditions. Accordingly, the thesis is structured into the following distinct sections:

- It is suggested that one challenge in the evolution of combinatorial systems is **overcoming iconicity**, where signs or gestures possess inherent, specific meanings that resist free recombination. Iconicity offers an alternative solution to problems related to learning and transmission that arise in cultural evolution. The first study tests the hypothesis that iconicity and combinatorial structure might conflict.

- The second section hypothesizes that combinatorial structure emerges from balancing simplicity and informativeness, placing emphasis on the the **role of communicative efficiency**, alongside the established role of learning bottlenecks in the emergence of structure. This offers insights into the specific conditions under which combinatoriality might arise in human culture.

- The final section addresses the **computational basis** for the human capacity for combinatorial structure. It elaborates on previous speculations about simplicity-focused inductive biases that arise in human learning by leveraging advancements in probabilistic programming and machine learning. A neuro-symbolic computational model is developed, combining opposing approaches to concept learning and generalization. This provides evidence for the role of symbolic inductive biases in the emergence of structured symbol systems.

Taken together, these individual studies help motivate a broader cohesive vision: Combinatorial systems emerge when specific cognitive processes, underpinned by computational adaptations for symbolic processing, are amplified by social-cultural processes on timescales that extend beyond the individual learner (Smith, 2020). This synthesis not only addresses a fundamental question in the evolution of human communication, but also exemplifies broader patterns in cultural development and cognitive science (Mesoudi & Thornton, 2018).

Overcoming Iconicity

A central question in evolutionary linguistics is the relationship between iconicity and combinatoriality. Iconicity, where symbols reflect sensory-motor attributes of their referents, reflects a departure of the principle of arbitrariness central to linguistics (Mahowald, Dautriche, Gibson, & Piantadosi, 2018; De Saussure et al., 1916). Crucially, iconicity seems to conflict with combinatorial structure. The latter requires that symbols be formed using a small set of building blocks, conforming to syntactic rules rather than semantic properties, enabling the system to generate limitless expressions from a finite set of elements (Dingemanse, Blasi, Lupyan, Christiansen, & Monaghan, 2015; Roberts et al., 2015). Iconicity, in contrast, imposes constraints on the relationship between form and meaning. This can be challenging in a combinatorial system where symbols are constructed from a predefined set of building blocks, potentially limiting the ability to directly mirror relevant referent features.

Research further indicates that iconicity and combinatoriality may fulfill distinct roles. Iconicity is foundational for bootstrapping communication, particularly in the absence of established conventions. Studies show that in the early stages of language development, gestures, which have a stronger affinity for iconicity, are preferred over spoken forms due to their representational transparency (Fay et al., 2022).

Combinatoriality, on the other hand, seems to emerge for transmission efficiency, making messages themselves simpler to produce and perceive (Roberts et al., 2015). This suggests that signals initially start out iconic but will eventually become more arbitrary as they develop into combinatorial systems (Zuidema & de Boer, 2018). Verhoef, Kirby, and de Boer (2016) conducted an experiment involving the transmission of artificial languages across generations of participants, providing supporting evidence to this idea. The study found that the onset of combinatorial structure was delayed in a condition where forming iconic relationships was possible, but it was not possible to quantify the impact of iconicity directly.

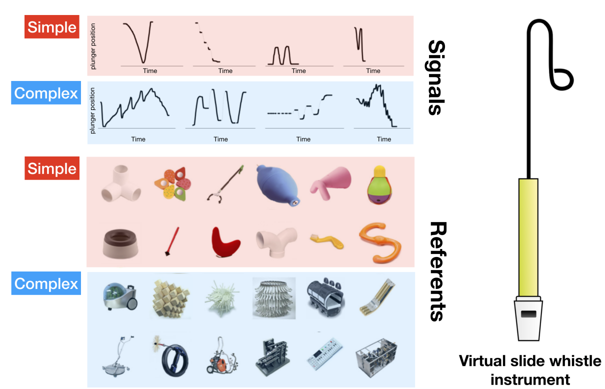

Figure 1: **Experimental stimuli.** Illustration of the stimuli and the virtual slide whistle instrument used in the iterated learning experiment. Whistled signals are displayed, with line trajectories indicating pitch, alongside visual referents, categorized as simple (red) or complex (blue). Initially, these were paired in an unsystematic manner so that languages would start out as non-iconic.

This chapter aims to bridge this research gap by **testing the hypothesis that iconicity and combinatorial structure are in tension** and providing quantitative evidence for their co-evolution. Additionally, the study focuses on a more abstract, systematic form of iconicity that exists in terms of the perceptual complexity of meanings and signals (Lewis & Frank, 2016).

To explore the simultaneous evolution of combinatoriality and iconicity, an iterated learning experiment (see Smith, Tamariz, & Kirby, 2013, for an overview

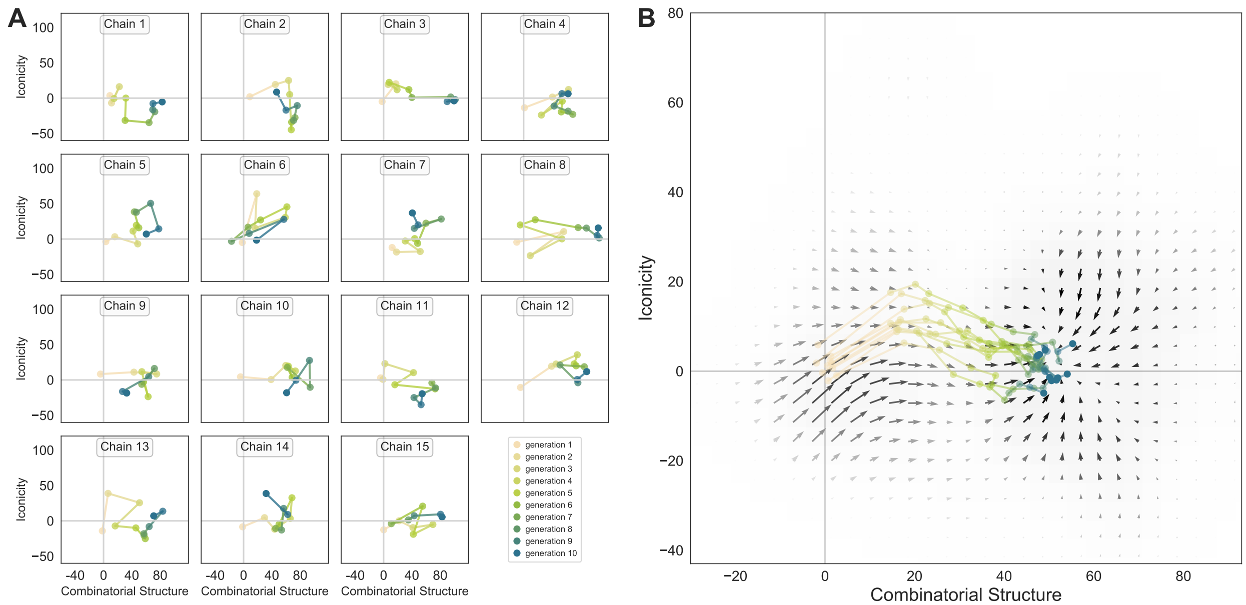

Figure 2: **Results from the experiment.** (A.) Evolutionary trajectories of transmission chains that were obtained through the iterated learning experiment, shown in terms of their iconicity and combinatorial structure. (B.) Using those trajectories, a vector field representation was constructed that shows transitions in a hypothesized language state space. Iconicity trends upwards when languages still lack combinatoriality, and otherwise tends back toward arbitrariness. The figure additionally shows novel trajectories obtained through simulations, illustrating the general trend transmission chains in this space tend to follow.

of the iterated learning paradigm) was designed using artificial languages comprised of whistled signals. This signaling space was chosen based on pioneering work by Verhoef, Kirby, and de Boer (2014). What makes this signaling modality particularly compelling is its similarity to fundamental aspects of the vocal-auditory modality in human speech, while simultaneously being a novel medium in which most people lack the same level of domain expertise as they have in language.

In the experiment, participants formed 'transmission chains' and learned whistled signals of varying complexity (illustrated in Figure 1), which initially contained no combinatorial elements. Signals were paired with visual referents of different complexity in a non-systematic, fully arbitrary way. Each participant then transmitted the language to the next in line, who acquired it in the same way they did. The languages underwent evolution over 10 generations. The development of combinatoriality and iconicity was then evaluated on the basis of a additional experiments, assessing complexity of the evolved signals and languages as a whole.

Results indicated a gradual emergence of combinatorial structure across multiple generations, while iconicity was rapidly incorporated but eventually diminished to levels indistinguishable from arbitrariness. This trend is depicted in Figure 2, presenting a vector-based representation of how these two properties evolved in relation to each other.

This decline aligns with the 'conflict hypothesis' outlined above, while also presenting a challenge that demands further research. As predicted under the account, iconicity diminishes as combinatorial structure continues to develop. This could be attributed to the favorable conditions for combinatorial structure in this setting, where subjects are required to encode, recall and accurately transmit a set of arbitrary and diverse signals, especially considering the absence of actual communicative use of these signals.

On the other hand, the disappearance of this more abstract form of iconicity, characterized by complexity relations, is intriguing. Theoretically, such a form of iconicity could coexist with different combinatorial structures, making its disappearance unexpected and challenging the mere conflict account. The loss of iconicity, even in forms potentially compatible with combinatorial structures, suggests a more nuanced

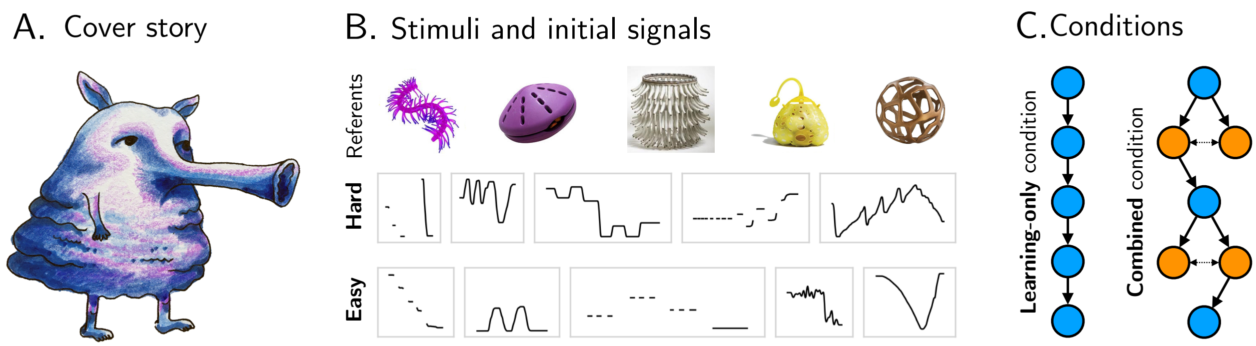

Figure 3: **Stimuli for the experiment contrasting the role of communication and transmission.** (A.) In the cover story, subjects were told that they learn signals produced by aliens on planet Vuma, who communicate using their whistle-like trunk. (B.) Data for two separate initial conditions was collected, where either five individually simpler or five slightly more complex signals were paired with novel visual referents taken from (Lewis & Frank, 2016). The assignment of referents to signals was randomized across chains and between successive generations. (C.) The two different experimental conditions.

interplay among the underlying cognitive biases and resulting preferences for iconicity or arbitrariness at different stages in evolution.

Balancing Simplicity and Communicative Efficiency

The second part of the thesis shifts focus from iconicity to the mechanisms behind the emergence of combinatorial structure. It aims to explain its emergence as a result of conflicting forces acting cultural evolution, an idea with historical roots in linguistics (Zipf, 1949; Martinet, 1952; Hockett, 1960) and recent grounding in mathematical accounts building on information theory (Plotkin & Nowak, 2000; Gibson et al., 2019).

Linguistic structure is thought to emerge from trade-offs between simplicity and informativeness, amplified across generations through cultural transmission. Learning pressures favor simpler languages (Smith, Kirby, & Brighton, 2003). However, overly simplistic languages risk losing communicative utility. To maintain informativeness, languages must balance simplicity with the need to make clear distinctions, effectively limiting excessive simplification (Spike, Stadler, Kirby, & Smith, 2017). Combinatorial structure seems to offer a solution to this balance, an idea that has recently been corroborated through a computational model presented in Kirby and Tamariz (2022). By reusing elements in systematic ways, languages can remain simple (easier to learn and use) yet expressive (capable of conveying diverse messages). This suggests combinatorial systems are preferred over holistic ones, as they align better with human learning biases, making them more efficient.

This chapter aims to empirically test the hypothesis that **combinatorial structure arises as a trade-off between simplicity and informativeness**. An online multiplayer experiment was developed which expands on the iterated learning framework introduced in the first chapter. This is done by incorporating a communicative task, allowing the examination of signaling systems under simultaneous pressures for simplicity and informativeness organically arising from the communicative task. The experiment, following a standard iterated learning procedure, had two conditions: a learning-only condition, where subjects learned and transmitted miniature signaling systems, and a combined condition that added a communication game to the learning phase (see Figure 3). The same whistled signal modality is used as in the previous set of studies.

The study revealed that only languages exposed to both learning and communication evolved to exhibit system-wide combinatoriality, as depicted in the trajectory labeled 'learning + use' in Figure 4. Conversely, languages subjected to the learning-only condition underwent simplification due to learning constraints. However, without the element of communication, this simplification compromised communicative utility, resulting in non-informative systems like the one shown in the bottom left corner of Figure 4. Communicative thus played a crucial role in allowing the reduction of complexity without compromising signal discriminabil-

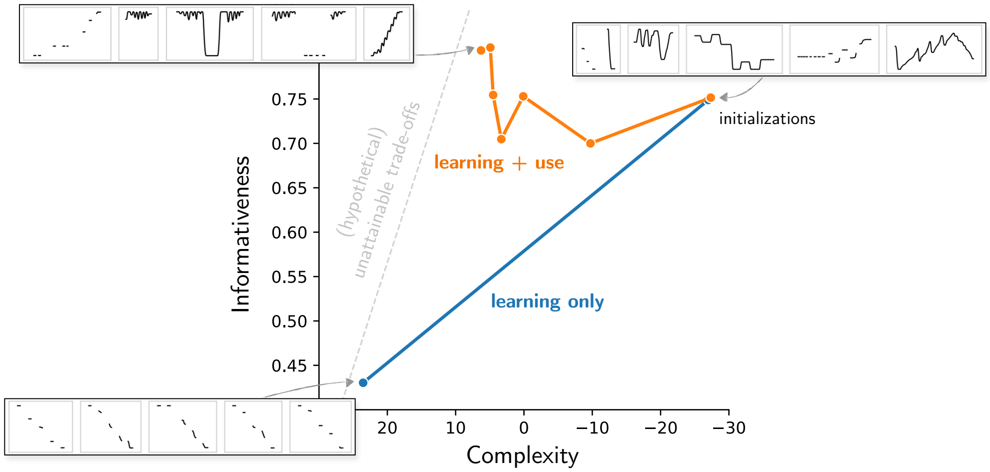

Figure 4: **Results showing simplicity informativeness trade-offs.** Results visualizing trade-offs between simplicity and informativeness. Languages start out as both complex and expressive (example language in the top right corner). Languages in the combined condition increase in communicative utility and decrease in complexity. This reduction in complexity is achieved through the utilization of combinatorial building blocks, as exemplified by the top left language in the figure. Languages in the learning-only condition become even simpler over time, but do so at the expense of loosing discriminability (see the example language in the bottom left corner).

ity, leading to informative systems adapted for efficient communication. It also facilitated the alignment of individual combinatorial patterns into broader, shared conventions, enhancing communication performance.

These findings support to the trade-off hypothesis, stating that combinatorial structure may arise as an adaptive solution to the trade-off between simplicity and informativeness, arising from the interplay of cognitive biases with functional pressures, rather than being an inherent characteristic of symbolic systems. In short, combinatorial systems are simple due to element reuse, yet informative due to the diverse signals created by rearranging these elements. Consequently, this insight also helps resolve the puzzle stated in the introduction, providing clarity on why combinatorial structure it is not as ubiquitous in human culture as one might expect.

The Computational Basis of Combinatorial Structure

A recurring theme in the study of combinatorial behavior in humans is the mind's propensity for finding patterns in data, even when that structure is not really there. This is particularly well illustrated in the spontaneous development of structure in home sign systems by deaf children and in the gradual progression from holistic to systematic gestures in emerging sign languages such as Al-Sayyid Bedouin Sign Language (ABSL) (Senghas & Kita, 2004; Aronoff, Meir, Padden, & Sandler, 2008; Sandler, Aronoff, Meir, & Padden, 2011). In light of these real-world examples, along with the experimental studies presented earlier, the final chapter of this thesis leverages computational modeling to delve deeper into these cognitive processes. This shift in focus aims to precisely articulate the learning constraints and computational mechanisms at play and how they facilitate the development and evolution of combinatorial symbol systems.

What mechanisms in the human mind enable it to deal with combinatorial structure? These likely include the ability to detect and extract building blocks (Harnad, 2003; Goldstone & Hendrickson, 2010), dealing with discrete elements in the context of sequential structures (Goldwater, Griffiths, & Johnson, 2009; Elsner, Antetomaso, & Feldman, 2016), maintaining representations about a domain on multiple levels of abstractions (Kemp, Perfors, & Tenenbaum, 2007; Feldman, Griffiths, Goldwater, & Morgan, 2013), and domain-general preferences for simplicity (Chater & Vitányi, 2003).

However, understanding how these processes function together to enable learners to learn and bootstrap structure remains a challenge. To address this, a neuro-symbolic generative model is introduced that aims to

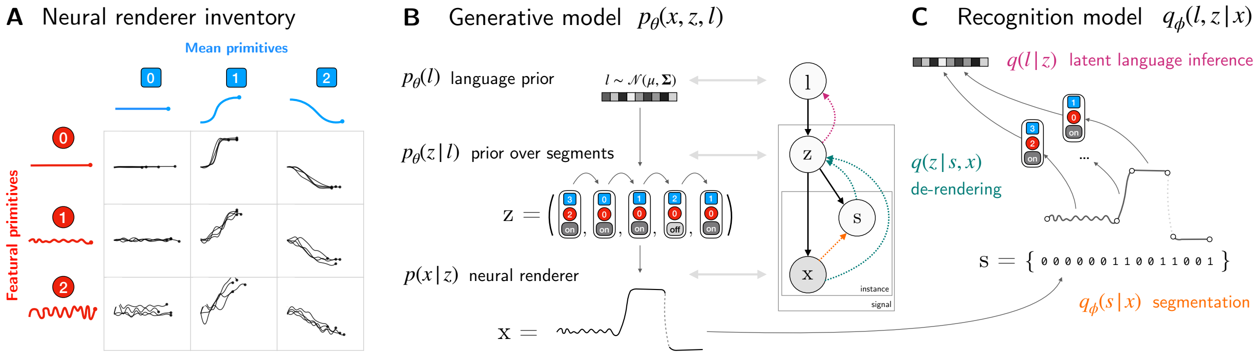

Figure 5: **Neuro-symbolic generative model.** (A.) A neural renderer, which consists of an inventory of mean and featural primitives that compose into rendered signal parts. (B.) Starting from these primitives, signals in the generative model are composed using a prior over segments $z \sim p_\theta(z)$ and a neural renderer $p(xz)$ which produces the continuous signal x. The hierarchical model additionally consists of a global latent code ℓ which parameterizes the bigram conditional prior $p_\theta(z\ell)$. (D.) The recognition model used for inference is decomposed into *segmentation* $q_\phi(sx)$ (either filtering- or neural network-based) and derendering $q(zs, x)$.

explore these factors within a single computational framework. Inspiration is drawn from work in the domain of concept learning, where various model classes have been proposed, ranging from structured probabilistic models (Lake, Salakhutdinov, & Tenenbaum, 2015), notable for their precision in causal and compositional understanding of data, to neural network models (Ganin, Kulkarni, Babuschkin, Eslami, & Vinyals, 2018), celebrated for their adaptability and scalability. The neuro-symbolic generative model presented in this chapter seeks to integrate the robustness and generalizability of symbolic methods with the adaptability inherent in neural models, especially in continuous data domains such as the ones explored in this thesis (see Feinman & Lake, 2020, for a related approach). By doing so, it seeks to provide a comprehensive computational account on how humans learn and ultimately evolve combinatorial systems.

Our neuro-symbolic model treats signals as sequences of discrete segments, akin to linguistic phonemes, and uses neural networks to render these in a continuous signal-transmission modality (see Figure 5 for modeling details). This approach allows for systematic generalization typical of symbolic models, while also handling continuous perceptual data.

To test how different cognitive biases instantiated in the model contribute to behavior, models are compared to neural network-only architectures, such as recurrent neural networks and transformer models, focusing on their ability to learn and generalize from whistled signal data obtained from previous experiments discussed in earlier chapters. Concretely, models are evaluated in their ability to recognize of individual signals, and to identify signals as part of a specific language system in which they have evolved. In addition to the computational models, a group of naive human subjects is recruited to perform the same set of tasks. This human benchmark serves as a crucial reference point, offering insights into how the model's performance aligns with human cognitive abilities in processing and interpreting combinatorial signal systems.

Results from this demonstrate that the neuro-symbolic model outperforms neural network-only models in learning and generalizing the whistled signal data, and also behaves more human like in terms of the errors it makes (see Figure 6 for the complete set of results across all three tasks). This suggests that the human ability to develop combinatorial systems may be rooted in a blend of symbolic reasoning and statistical pattern recognition as instantiated by the neural parts of the hybrid neuro-symbolic model.

The implications of this research extend beyond linguistics to broader cognitive science domains. Understanding the cognitive mechanisms that underlie the acquisition and evolution of knowledge domains such as language that seem to be characterized by rich and systematic causal relationships but are grounded in perception is key to comprehending human intelligence. The model explored in this section provides insights into how such concepts could be acquired from raw data, which constitutes a broader challenge in current machine learning systems. The evolution of symbolic systems from raw continuous data, as examined in this thesis, could provide a fertile ground for further research in this direction.

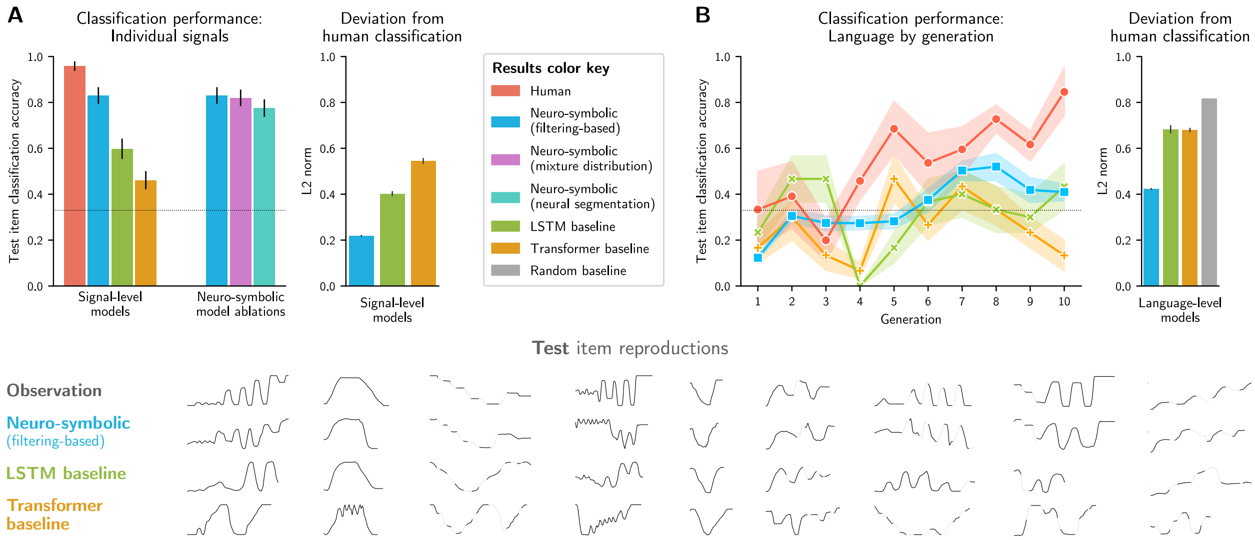

Figure 6: **Results comparing models with each other and with human performance.** (A.) The neurosymbolic model classifies test set items more accurately and its performance is more human-like than either neural baseline. There is no significant difference in performance between the different inference models. (B.) In language classification, our hierarchical neurosymbolic model captures human performance better than the two neural baseline models. Like in the human subject experiment, neurosymbolic classification accuracy increases as a function of generation, indicating that the model successfully picks up combinatorial structures from more mature languages. (C.) In terms of signal reproduction (bottom figure) the neurosymbolic model variants consistently outperform their neural competitors on signals not included in the training set. This suggests that the neurosymbolic model is better at handling out-of-distribution data, showcasing a stronger generalization capability.

Conclusion

One central questions in the cognitive sciences is to understand the origins and development of combinatorial symbol systems in human culture, enabling diverse forms of communication, collaboration and creativity. Among important outstanding questions (Zuidema & de Boer, 2018), this thesis has specifically focused on the social and cultural processes that might allow these systems to evolve, as well as the cognitive capacities that enable us to acquire and use them effectively. By combining novel behavioral and computational evidence, the thesis offers insights into how such systems could have evolved. These results also shed light on why combinatoriality, despite its apparent utility in communication and representation, might be so rare in natural systems:

- **Overcoming iconicity.** The initial study demonstrates a natural progression from iconicity to combinatorial systems, suggesting that the two might be in conflict. The ready adoption of iconicity, even though it was totally absent in the input, suggests that combinatorial structure might not be immediately prevalent or may not evolve at all in contexts where the benefits iconicity outweigh those of combinatoriality.

- **The role of communicative efficiency.** The second study provides key evidence that combinatorial systems likely emerge only under conditions where simplicity and informativeness (among other factors) are optimally balanced, as demonstrated through a cultural transmission experiment that incorporates a reference game.

- **The computational basis of combinatoriality.** Computational modeling in the final part of the thesis reveals the cognitive biases necessary for a learner to develop combinatorial structures. Beyond a mere preference for simplicity, it suggests that learners must possess the capacity to analyze and represent inputs symbolically. This requirement could explain the scarcity of combinatorial structures in other species lacking such sophisticated symbolic processing abilities.

The thesis revisits key themes such as the tension between iconicity and combinatorial structure, the role of meaning and compositionality in symbol systems, and the hierarchical nature of signaling systems. And while the thesis makes significant contributions to our understanding of symbol systems, several important limitations need to be acknowledged, such as the use of a simplified auditory signaling space and the potential influence of pre-existing knowledge of combinatorial patterns, among others. Future research should explore the emergence of combinatorial systems in more diverse modalities (Little, Eryılmaz, & de Boer, 2017) and in more naturalistic and complex settings (Morin et al., 2018). Additionally, while focusing on human symbol systems, the thesis recognizes that combinatorial signaling is not unique to humans. Future studies should compare human capabilities with those of other species (Engesser & Townsend, 2019) and examine the influence of social and cultural factors, such as community size and structure (Kirby & Tamariz, 2022), on the evolution of combinatoriality.

Understanding these aspects will further elucidate the unique position of human combinatorial structures in the broader tapestry of communication and cognition.

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